

Internal ballistic simulation of multi-burning-rate solid rocket motor based on parameterized feature CAD model

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Abstract: Internal ballistic simulation (IBS) method of multi-burning-rate solid rocket motor (SRM) was developed based on 3-D burning regression method by parameterized feature CAD model (PFCADM) and lumped parameter, in consideration of time-dependent, erosive-burning-effect from internal ballistic numerical algorithm. By driving multi-parameter CAD model based on PFCADM, the approach is capable of conducting the geometric regression simulation of various grain combinations of complex configurations with different burning rates. Through suitably simplifying the internal ballistic numerical algorithm, the problems of coupling geometric regression simulation of sub-grains of different burning rates and high computational consumption of internal ballistic calculation were solved. One tri-burning-rate grain motor, which had been firing-tested, was used as the validation case of simulation. The results show that, with the 3-D grain regression model and sufficient accurate internal ballistic algorithm, the method realizes IBS of the case in low computational-consumption prediction of its performance within the accuracy of 2% during 1 h clock-time. The application of the method provides a practical approach to aid SRM design of multi-burning-rate grain.

Key words: solid rocket motor; internal ballistic simulation; multi-burning-rate; grain design; unsteady flow

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Solid rocket motors (SRMs) of multi-burning-rate grain provide a more effective delivery of impulse than those with single-burning-rate grain and are therefore frequently specified for tactical missiles^[1]. On the other side, the increase of internal ballistic prediction capabilities can improve the SRM design reliability and reduce the development costs related to experimental activities and static firing tests^[2]. Therefore, an approach of accurate and low computational-consumption internal ballistic simulation (IBS) of complex multi-burning-rate SRM is worth exploring.

Most prior researches have mainly focused on the IBS of single- and dual-burning-rate pro-

pellants^[3-5]. Analytic geometric method^[6-8] with 0-D and quasi-steady internal ballistics model is one of the most common methods because of low computational consumption and high precision. Nevertheless, the grain modelling is only applicable for several specific grain configurations. For complex combination of different 2-D or 3-D grain configurations in the multi-burning-rate SRM, the geometric mathematical formulae are too complicated to derive due to the couplings between different configurations. In order to solve the limitations of the analytic geometric method, several non-analytic methods of grain burn-back simulation are developed for IBS with 0-D or 1-D flow field model. Willcox, et al^[2,9]

develop minimum distance function in the IBS program Rocballist. Frederic, et al^[4,10] use mesh burning-back simulation approach in the IBS programs of SNPE propulsion code and PIBAL. Yildirim, et al^[11] develop grain burn-back simulation program using level-set method^[12]. Cavallini, et al^[13-14] realize coupling quasi-steady 1-D flow field with level set burn-back method in the IBS program SPINBALL. The notable characteristic of these non-analytic methods is that the computational consumption increases rapidly with the complexity of grain configurations and number of propellant types if the simulation accuracy assured. Therefore, these methods are difficult to be applied in the IBS of multi-burning-rate SRM within acceptable computational efficiency, considering its complex grain configurations and interactions between different propellants. A simulation method of grain burning regression is studied based on CAD model with CAD technology developing^[15-17]. It has extensive adaptability in various complex grain configurations. Based on it, the IBS of single-burning-rate SRM with 0-D and quasi-steady flow field needs low computational consumption with accuracy assured. It provides an appropriate direction to solve the IBS problem of multi-burning-rate SRM.

Considering the complexity of IBS for multi-burning-rate SRM, three aspects need to be addressed:

- 1) Explore simulation method of grain regression which is capable to adapt kinds of 2-D and 3-D complex grain configurations, support simultaneous regression simulation of different burning-rate sub-grains, and deal with the interactions of grain regression between them.

- 2) Rationally simplify the internal ballistic numerical algorithm to reduce the computational cost on the basis of ensuring the accuracy of calculation.

- 3) Choose appropriate simulation scheme to realize the coupling simulation of grain regression and internal ballistics.

The objective of this paper is, hence, to develop a simulation capability of internal ballistics for multi-burning-rate SRM from building-up of

pressure in chamber to burn-out of the grain with simplified internal ballistic algorithm and accurate 3-D grain geometric regression model, in order to predict the performance of multi-burning-rate SRM in acceptable precision, meanwhile, increasing the computational efficiency as much as possible.

1 Methodology of simulation

1.1 Internal ballistic model

Nowadays, internal ballistic models of single-burning-rate SRM mostly adopt 1-D flow field model^[2,13] or quasi-steady 0-D internal ballistic model^[4,18]. Considering the complexity of burning surface regression and gas property in multi-burning-rate motor, the quasi-steady 0-D internal ballistic model is relatively more suited in the IBS of multi-burning-rate motor to ensure its versatility to different types of grain configurations and high computational efficiency. Nevertheless, most 0-D internal ballistic models used in engineering fields completely neglect erosive burning effect and time-dependent effect, which result in large errors of internal ballistics.

The impacts of erosive burning on the SRM internal ballistics are two-fold: initial pressure peak and non-uniform burning-rate on the burning surface. Non-uniform burning-rate burning surface needs simulation coupled with the cross-flow velocity, which needs huge computational amount, which increases with the complexity of grain configuration and the number of propellants in multi-burning-rate SRM. Moreover, the regression simulation of non-uniform burning surface should be modeled for specific grain configurations and hard to maintain the wide applicability of the IBS approach. Hence, the regression simulation based on uniform burning-rate approximation is adopted to simplify computation in many IBS approaches^[4,19], of which the simulation results have a good agreement with experiment. On the other side, compared with the non-uniform burning-rate effect, the initial pressure peak influences the working stability of SRM and is more concerned in engineering

fields. The prediction of initial peak needs less additional computational consumption by applying erosive burning correction^[20] on the burning rate of each sub-grain.

Meanwhile, the initial peak and transition processes of multiply pressure stages, which are common in multi-burning-rate SRM, are greatly affected by the time-dependent effect.

A good trade-off between the stated needs for the numerical simulation of the overall burning time can be, therefore, the use of unsteady lumped parameter flow field model with erosive burning correction and uniform burning-rate approximation to ensure the calculation precision and increase the computational efficiency, while capable to deal with internal ballistic prediction of various combinations of grain configurations.

1.2 Regression simulation method

Solid modeling approach^[15-17] is an appropriate way of regression simulation to adapt various complex grain configurations, through relatively simple modeling and low computational consumption, meanwhile, keeping a high accuracy of simulation. Based on it, parameterized feature CAD model (PFCADM) approach is studied to solve the problem of simultaneous simulation of different burning-rate propellants. Namely, the regression parameters are embedded in the dimensions and constraints in CAD modeling process of the whole grain, and then by driving these parameters respectively, the simultaneous regression evolution of the sub-grains of different complex grain configurations and different burning rates is achieved. As for the interaction problem of different burning-rate sub-grains, the interaction burning surface can be handling by embedding related regression parameters in their dimensions following regression law by PFCADM approach.

Based on the uniform burning-rate approximation, the burning surface regression of all sub-grains follows the parallel geometric burning law. In the other words, the burning is assumed along the normal direction of the local burning

surface and the burning rates of burning surfaces of each sub-grain are all the same at the same time.

Comprehensive IBS of multi-burning-rate SRM is achieved by coupling the regressing simulation method and internal ballistic model.

2 Burning surface regression simulation approach based on PFCADM

2.1 Modeling of PFCADM

Based on the commercial CAD software, the PFCADM of each single-burning-rate grain can be conducted by the following steps:

1) According to the initial shape of the grain, analyze and divide the shape of the grain to several basic features. These features can be created by solid modeling operations, such as extruding, revolving, scanning, and so on. Then build the CAD solid model of the grain through creating the features and performing Boolean operations, such as join, cut, and intersection between them.

2) Parameterize the features of CAD model. That is, following parallel regression law of grain, analyze the changing process of each feature during the regression and replace some diameters and diameter constraints of the features by relation formulas defined with regression parameter, which is equivalent to the burned web of the grain.

A 2-D star core configuration grain is taken as an example shown in Fig. 1, where a, b, n, s_0 ,

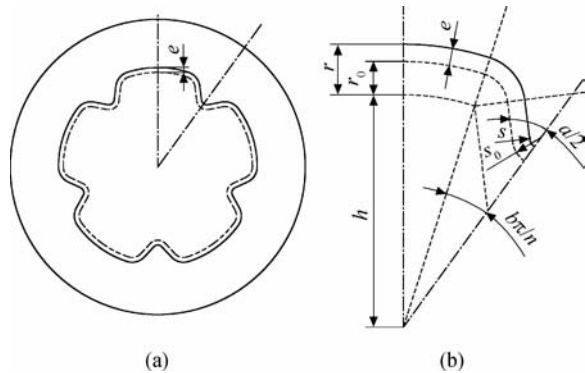


Fig. 1 Modelling example of 2-D star core configuration grain

h, r_0 are the geometric parameters of star configuration, and s, e, r are the geometric control parameters of burning surface. The first step of creating PFCADM is defining the shape of half lobe as a feature, which is constrained by the dimensions shown in Fig. 1(b). Using CAD tools, such as arraying, extruding and cutting, in the CAD software. The feature is transformed to the initial shape of star core configuration as shown in Fig. 1(a). The second step is, according to the parallel regression law of star core configuration, replacing the diameter r and s with relations of $r=r_0+e$ and $s=s_0-e$, where e is the regression parameter equivalent to the burned web. After that, the PFCADM of star core configuration for regression simulation preparation is completed.

Because the grain regression process of a given type of configuration is fixed, through establishing the PFCADM template library of various configurations, rapid grain modeling of complex combination of configurations is achieved by importing corresponding templates and customizing their dimensions.

2.2 Burning surface regression simulation method

Based on the PFCADM, the regression evolution of the single-burning-rate grain is implemented by driving the regression parameter to vary as the burned web. After each step of regression evolution, the geometrical analysis can be done by using CAD software module to obtain the geometrical information such as burning area, volume, etc. Fig. 2 presents the flowchart of

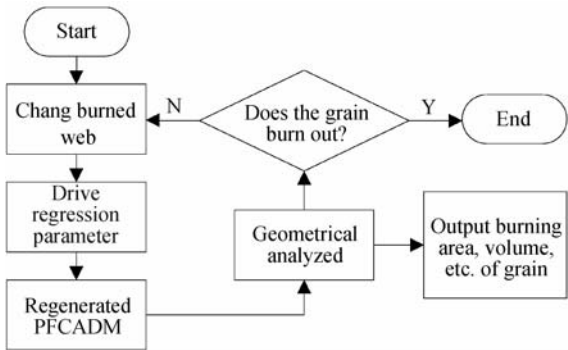


Fig. 2 Regression evolution process of the single-burning-rate grain based on PFCADM

the regression evolution process of the single-burning-rate grain based on PFCADM.

The modeling and regression simulation process of multi-burning-rate grain is essentially the same as one above of single-burning-rate grain. In order to realize the simultaneous regression simulation of all sub-grains, several changes of the process need to be made:

- 1) All sub-grains of different burning-rates are modeled respectively, while only in one same CAD model.
- 2) Each sub-grain is driven by one independent regression parameter respectively.
- 3) The regression simulation of multi-burning-rate grain proceeds by driving all the regression parameters simultaneously with the burned web of each sub-grain.

2.3 New generating burning surface problem

Distinguished with the simulation of single-burning-rate grain, the interaction problem of regression simulation between adjacent sub-grains of different burning-rates like new generating burning surface (NGBS) problem arises in the simulation of multi-burning-rate grain.

As shown in Fig. 3, it is a typical dual-burning-rate grain of tube configuration. Sub-grain 1 and sub-grain 2 are of high and low burning-rates, respectively. As the burning going, at the interface of the two sub-grains, NGBS is generated and regressed because of the difference of burning-rates. The process of NGBS regression depends on the burned webs of both sub-grains.

According to the law of parallel regressing, the NGBS is a curved conical surface and the curved angle lies on the burning rate ratio of the two sub-grains. As shown in partial enlargement sub-figure A' of Fig. 3, the curved angle α varies along the curved surface as the burning rate ratio of the two sub-grains changes from initial to current time. Because the burning direction of each point needs satisfying that the components NGBS burning rate vector, along both normal directions of the two adjacent burnings surfaces, equal to the burning rates of two sub-grains re-

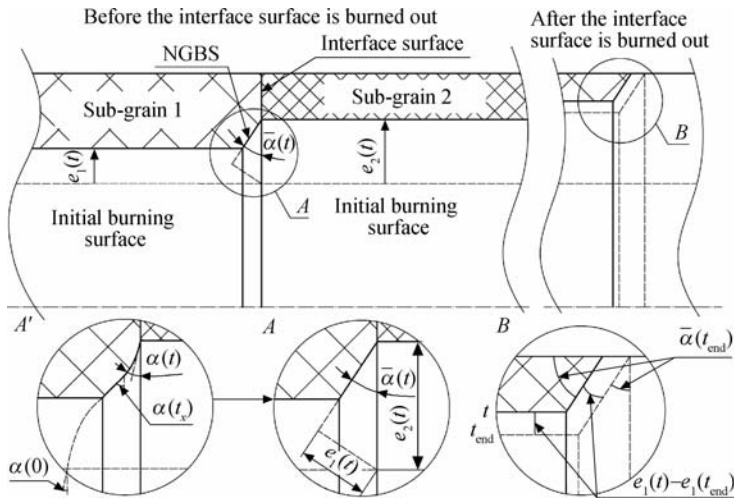


Fig. 3 Sketch of NGBS problem

spectively,

$$\tau_{12}(t_x) = \sin[\alpha(t_x)] = \frac{r_1(t_x)}{r_2(t_x)} = \frac{a_1 [p_c(t_x)]^{n_1 - n_2}}{a_2} \quad (1)$$

From Eq. (1), the sensitivity of burning-rate-ratio $\tau_{12}(t_x)$ to the chamber pressure p_c can be derived, where $r_1(t_x)$ and $r_2(t_x)$ are the burning rates of sub-grain 1 and sub-grain 2 of time t_x , respectively, a_1, n_1 and a_2, n_2 are their coefficients and pressure exponents of burning rates, respectively:

$$\frac{d\tau_{12}}{\tau_{12}} = (n_1 - n_2) \frac{dp_c}{p_c} \quad (2)$$

Equation (2) reveals that proportional changes between burning-rate-ratio and chamber pressure have linear correlation, and the impact of chamber pressure changing on the changing of burning-rate ratio mainly depends on the difference between their burning-rate exponents. In most multi-burning-rate grain designs, the difference of burning rate pressure exponents between propellants, generally, doesn't exceed 0.1 in order to keep internal ballistic stability of each thrust stage. During the stable stages of internal ballistic, the impact of chamber pressure changing on the burning-rate ratio can be neglected. Therefore, the angle α of NGBS at each place can be considered, approximately, identical during these periods, and the NGBS can be

considered as non-curved conical surface.

Assuming the ratio of burning rates of the two sub-grains remains constant, the average angle $\bar{\alpha}(t)$ between NGBS and interface surfaces of these two sub-grains meets the relation as below:

$$\sin[\bar{\alpha}(t)] = \int_0^t \frac{r_1(t_x)}{r_2(t_x)} dt_x \approx \frac{\int_0^t r_1(t_x) dt_x}{\int_0^t r_2(t_x) dt_x} = \frac{e_1(t)}{e_2(t)} \quad (3)$$

where e_1 and e_2 are the burned webs or the regression parameters of sub-grain 1 and sub-grain 2, respectively.

After the interface surface is burned out, the evolution of NGBS is not influenced by the burning rate of sub-grain 2 anymore. Then, NGBS is evolved like other burning surfaces conforming the law of parallel regressing. In the example of Fig. 3, the angle of NGBS to the interface surface will keep to the angle of NGBS at the end time of interface surface burning out $\bar{\alpha}(t_{\text{end}})$, until NGBS is totally burned out.

The regression simulation of NGBS is realized easily using PFCADM. Taking the NGBS of Fig. 3 as an example, NGBS should be constructed as a new feature during the modeling of sub-grain 1. Distinguished with features of other burning surfaces, the relation of Eq. (3), which is embedded with the regression parameters of e_1 and e_2 , should be imposed into the CAD model

feature of NGBS as a diameter constraint. Before the interface surface is burned out, t in the constraint relation equation is assigned with current simulation time. And when burning out of interface surface is detected, t is fixed to the constant t_{end} and NGBS will be driven by $e_2(t)$ to be evolved like other burning surfaces of sub-grain 1.

In addition, there is one infrequent circumstance that pressure stage of internal ballistic is switched to another one leading to significant variation of chamber pressure, due to the design of grain or one other sub-grain is burned out, before the interface surface is not burned out. On this case, the approximation of NGBS evolving using Eq. (3) would produce significant error. A good solution based on PFCADM is easy to be achieved by separating it to two NGBS features corresponding to the two pressure stages, respectively. In the prior pressure stage, the first NGBS feature is generated and evolved. After the pressure stage is switched, the second NGBS feature is generated and evolved starting from the ending point of interface surface of first stage. At the same time, the first NGBS feature is evolved following the parallel evolving law with the fixed angle $\alpha(t_{\text{switch}})$, where t_{switch} is the switching time of pressure stages. Furthermore, it is easily extended to the circumstance of more pressure stage switches during NGBS evolving.

Moreover, as to more complex grain configurations, NGBS features could be constructed only needing conformed to the NGBS evolving rules above at each vertical cross section to the interface surface, using CAD tools such as sweeping, blending, etc. when constructing the grain configuration features.

3 Internal ballistic model

3.1 Lumped parameter model

For the characteristics of multi-burning-rate SRM, unsteady lumped parameter internal ballistic model is adopted. Several assumptions are given as below:

1) The burning rate is uniform on the burning surfaces of each sub-grain and all the burning

surfaces are ignited at the same time when the chamber pressure reaches the ignition pressure.

2) The gas properties are all the same everywhere in the chamber and the mixture gas is a non-reactive and adiabatic perfect gas.

3) The erosion of nozzle throat is considered as linear changing.

4) In the mixing process of gases from different propellants, the thermodynamic properties except the gas temperature and the gas molecular weight are the same. The mixing process is isentropic and transient, namely, neglecting the duration of gases mixing process from different propellant.

Based on the assumptions and the law of mass conservation^[21], the internal ballistic partial equation of multi-burning-rate SRM could be derived:

$$\frac{V_c}{\Gamma^2 c^{*2}} \cdot \frac{dp_c}{dt} = \sum_i \rho_{bi} r_i A_{bi} - \frac{p_c A_t}{c^*} \quad (4)$$

where the subscript i means the parameter of i -th sub-grain, the same below; p_c is the pressure of chamber; V_c is free volume of chamber; Γ is a function of specific heat ratio $k^{[21]}$. c^* , R and T_t are the characteristic velocity, the gas constant and the total temperature of mixture gas, respectively; ρ_b , A_b and r are the density, the burning area and the burning rate of sub-grain, respectively; $A_t = 1/4 \cdot \pi(d_{t0} + 2\gamma_d t)^2$ is the area of nozzle throat, considered as linear erosion during the working process, d_{t0} is the initial diameter of nozzle throat; γ_d is the erosive ratio of nozzle throat, which is estimated based on previous experiments.

In order to illuminate the influence of the time-dependent effect, here a simulation using quasi-steady 0-D internal ballistic model has been derived as a comparison with unsteady model:

$$\sum_i \rho_{bi} r_i A_{bi} = \frac{p_c A_t}{c^*} \quad (5)$$

In Eq. (4) and, Eq. (5), the characteristic velocity c^* of mixture gas cannot directly be obtained from thermodynamic calculation of propellant, because it depends on different propellants in the chamber. Based on 4th assumption,

$$c^* = \frac{\sqrt{T_t R}}{\Gamma} \quad c_i^* = \frac{\sqrt{T_{ti} R_i}}{\Gamma} \quad (6)$$

where R_i is the gas constant derived from gas molecular mass of i -th sub-grain propellant. The gas constant R of mixture gas depends on the average molecular mass, R is determined by

$$R = \frac{\sum_i \dot{m}_{bi} R_i}{\dot{m}_b} \quad (7)$$

where $\dot{m}_b$ is the mass flow rate of sub-grain propellant.

According to the energy conservation, the internal energy of mixture gas

$$U = c_v T_i \dot{m}_b = \sum_i U_i = \sum_i c_v T_{fi} \dot{m}_{bi} \quad (8)$$

where c_v is the specific heat capacity at constant volume.

Unite Eq. (6) to Eq. (8),

$$c^* = \frac{\sqrt{\left(\sum_i \dot{m}_{bi} c_i^{*2} / R_i\right) \left(\sum_i \dot{m}_{bi} R_i\right)}}{\sum_i \dot{m}_{bi}} = \frac{\sqrt{\left(\sum_i \rho_{bi} r_i A_{bi} c_i^{*2} / R_i\right) \left(\sum_i \rho_{bi} r_i A_{bi} R_i\right)}}{\sum_i \rho_{bi} r_i A_{bi}} \quad (9)$$

From Eq. (9), it can be found that the characteristic velocity c^* of mixture gas keeps changing over time with the burning rates and areas of sub-grains. Therefore, Eq. (9) needs to be introduced into the internal ballistic equations and the result of c^* can be obtained by solving the entire internal ballistic equations uniting Eq. (9) and Eq. (4).

3.2 Erosive burning correction of burning rates

The burning rate for each sub-grain is assumed as a term dependent on the pressure, given by the classical APN expression, and an erosive correction coefficient ϵ_i as correction;

$$r_i = \epsilon_i a_i p_c^{n_i} \quad (10)$$

For end burning surfaces, there is no erosive burning effect, therefore, $\epsilon_i = 1$. For side burning surfaces, ϵ_i depends on the crossover velocity of mixture gas near the burning surfaces. Here, to simplify the calculation, an widely-used empirical equation^[20] is employed to estimate the erosive burning correction coefficient

$$\epsilon_i = \begin{cases} 1 & \text{if } \chi < \chi_{\text{ex}} \\ 1 + \xi(\chi - \chi_{\text{ex}}) & \text{if } \chi \geq \chi_{\text{ex}} \end{cases} \quad (11)$$

$$\chi = \frac{S}{A} \quad (12)$$

where S is the sum of burning area upstream to the local cross section. A is the area of local cross section of port. ξ and χ_{ex} is the empirical parameters from practical engineering experience and experiments. Here, $\chi_{\text{ex}} \approx 100$, $\xi = 0.0035$.

3.3 Algorithm of numerical simulation

In consideration that the internal ballistic Eq. (4) is an ordinary differential equation, the chamber pressure is solved in a fourth-order Runge-Kutta time integration scheme^[22]. Therefore, transform Eq. (4) to $f(y, t)$ form

$$\begin{cases} y = p_c \\ f(y, t) = \frac{dp_c}{dt} = \left(\sum_i \rho_{bi} r_i A_{bi} - \frac{p_c A_t}{c^*} \right) \frac{\Gamma^2 c^{*2}}{V_c} \\ h = \Delta t \end{cases} \quad (13)$$

where Δt is the time step of IBS. The chamber pressure during the working period can be calculated by iterating Eq. (13) from building-up of pressure in chamber to the end of the SRM working period.

The IBS approach of multi-burning-rate grain SRM is realized by integrating the grain regression simulation of PFCADM and internal ballistic calculation. The overall process is presented in Fig. 4.

In Fig. 4, the serial numbers i , j and n are the ones of sub-grain, Runge-Kutta iteration step and time iteration step, respectively. In the “Runge-Kutta iteration” area, all the parameters without superscript refers to the ones of time step n for simplicity. $t_{\text{rk}(j)}$ and $p_{\text{rk}(j)}$ equal to the argument expressions in the function f of the corresponding equation in the Runge-Kutta four steps according to the number j . p_{end} is the end pressure of SRM working period.

As for the calculation of quasi-steady model, Eq. (5) can be solved directly using Newton-Raphson algorithm^[22].

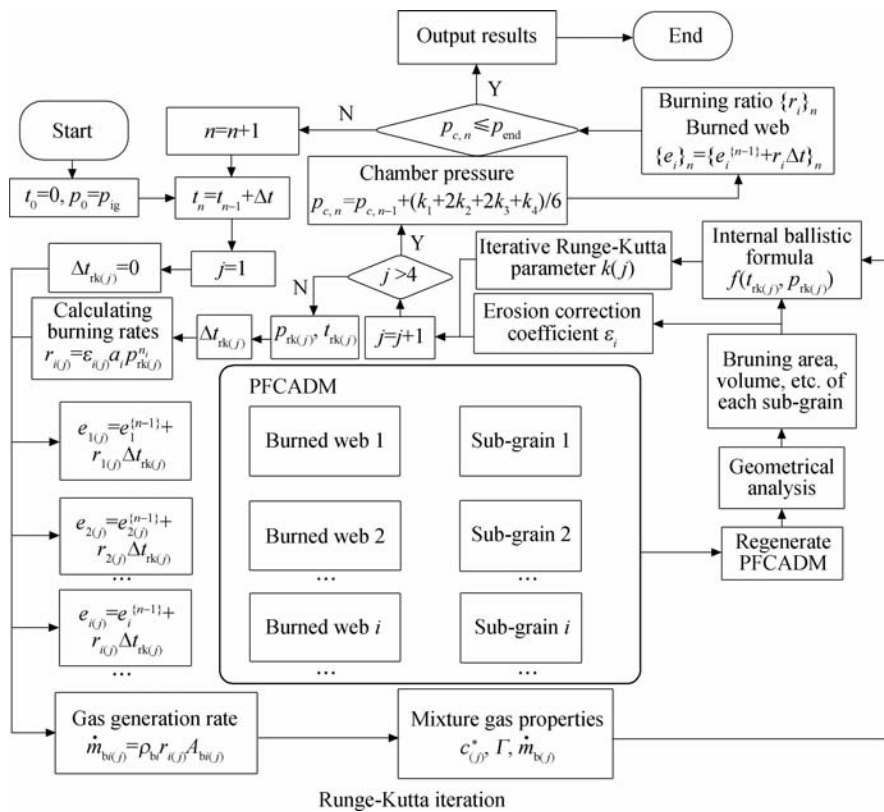


Fig. 4 Overall IBS process of multi-burning-rate SRM

4 Result and discussion

In order to validate the effectiveness of the approach, it is applied on the IBS of a tri-burning-rate grain SRM, which is actually designed and tested in a certain weapon system in order to reduce the highest pressure spike, as shown in Fig. 5.

The grain is combined with three segments of sub-grains in tandem. Sub-grain A of tube configuration is of low burning-rate propellant; sub-grain B combined of tube and star core configurations in series, at the junction of which there exists a more complex transition section

mixed with cone core and star core configurations, is of medium burning-rate propellant; sub-grain C of star core configuration is of high burning-rate propellant. Sub-grain B and sub-grain C is cohered together, and their cohered surfaces are identical. The burning surfaces consist of the all uncoated surfaces of the grain, which are illuminated in Fig. 5. The burning rate parameters are given in Table 1. In Table 1, corresponding units of the chamber pressure and the burning rate in APN formula are MPa and mm/s respectively.

The simulation starts with the opening of nozzle cover when the chamber ignition pressure

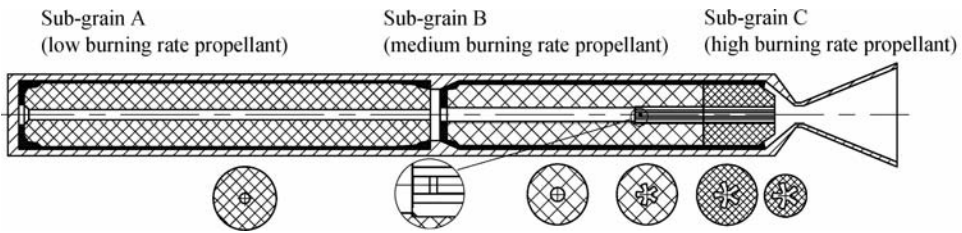


Fig. 5 Assembly structure and configurations of tri-burning-rate grain SRM

reaches 1.0 MPa, and ends when the chamber pressure is below 0.3 MPa. In order to compare the internal ballistic results more accurately, when calculating the thrust and specific impulse result of simulation, the thrust coefficient is corrected by the nozzle performance average loss coefficient $\bar{\eta}_{c_i}$ from experiment.

On the CAD software Pro/ENGINEER, the tri-burning-rate grain is modeling as PFCAMD consisting of 108 features. Based on it, after the 1 h computation of IBS on the computer of Intel Core Dual 2 GHz CPU with the simulation accuracy of time step as 0.01 s, 3-D burning surfaces evolution process is simulated as presented in Fig. 6. For the sake of illuminating the evolving process of NGBS, sub-grain C and sub-grain B are separated manually in the figure. The simulation result is presented in Fig. 7 and Table 2. In particular, the NGBS between sub-grain B and sub-grain C is generated and regressed reasonably.

Table 1 Burning rate parameters of propellants

Sub-grain	Burning-rate coefficient	Burning-rate exponent
Sub-grain A	3.90	0.40
Sub-grain B (<13 MPa)	5.25	0.40
Sub-grain B (>13 MPa)	5.05	0.41
Sub-grain C	12.13	0.36

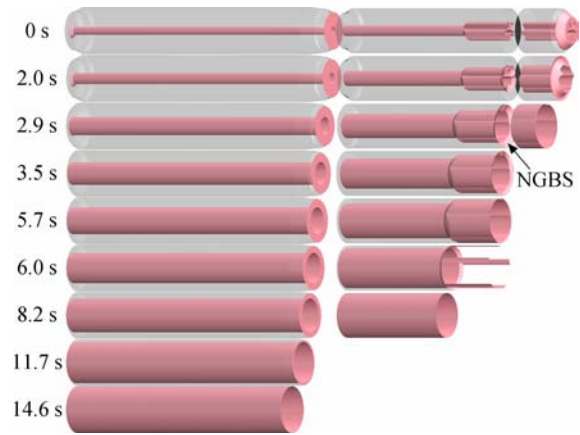


Fig. 6 Burning surface evolution process of the grain

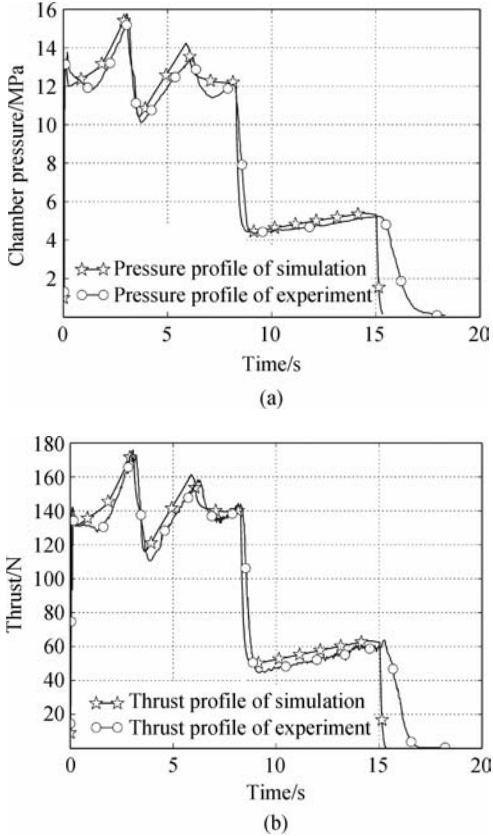


Fig. 7 Chamber pressure and thrust profiles of simulation and experiment

Table 2 Comparison result between simulation and experiment

Parameters	Error/%
Total impulse	-0.18
Specific impulse	-1.77
Highest pressure	+1.09
Propellant mass	+1.04
Working duration	-8.43

Based on PFCADM, the chamber pressure and thrust profiles of SRM in Fig. 7 show a good agreement in the most period of the working period between simulation and experiment results. The measured pressure trace of the motor shows a prominent peak just after ignition and it is captured accurately in the simulation by the effective burning correction and unsteady effect considered. Quantitative comparisons in Table 2 show that most errors of main performance parameters are below 2%.

In this example, the difference of burning

rate exponents between propellants is below 0.05. In other words, according to Eq. (2), if chamber pressure changes in 10%, burning rate ratio between two different propellants changes in 0.5% or less. Therefore, The error of NGBS approximation on the final simulation result could be neglected.

Although the simulation approach achieves good results in most working periods, the deviation still exists remarkably in the pressure-descending period. It probably arises from some differences between the ideal simulation model and the actual complicated tail-off phase of SRM, when close to burnout of all propellants, become greater, due to the limitation of the proposed approach.

The simulation without erosive burning correction of burning rates is conducted to analyze its influence upon the initial peak. The comparison of pressure profile is shown in the top figure of Fig. 8. The results show a much improved prediction of the initial pressure trend with the addition of erosive burning correction in simulation, and the erosive burning is the primary cause of initial peak. The results of Fig. 8 also confirm that the erosive burning contribution on the burning rates is small after the first pressure peak.

Because the erosive burning is typically most important early in the working period, the burning rates of all the sub-grains with and without erosive burning correction are computed and shown in Fig. 9. As expected, the erosive burning of sub-grain C at the downstream of the

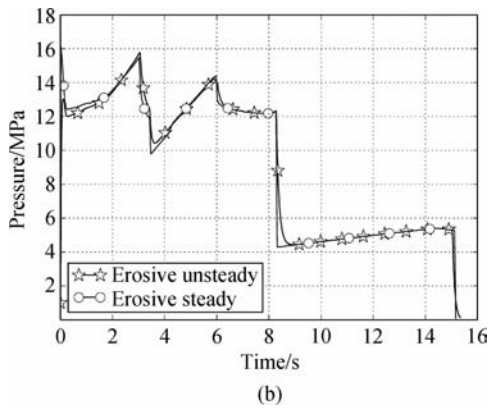
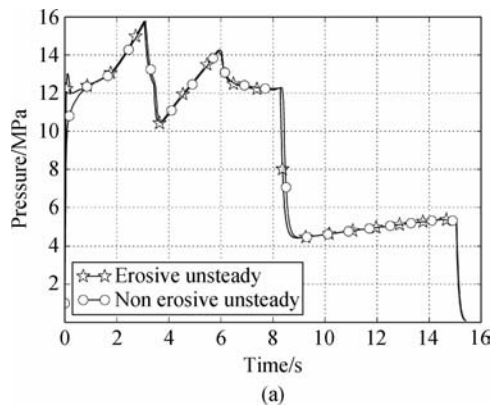
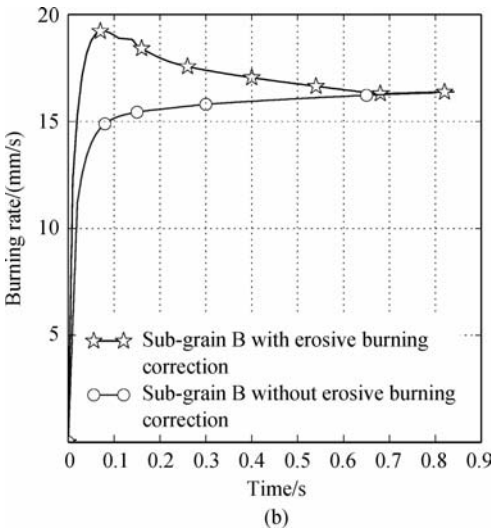
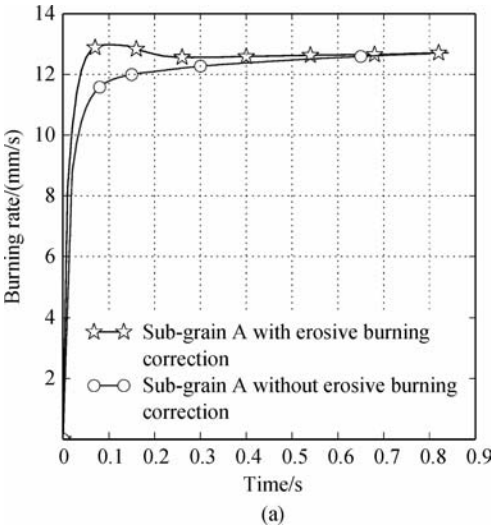


Fig. 8 Pressure profile comparisons with non-erosive simulation and quasi-steady simulation

grain is more severe than the other two sub-grains, while its duration is shorter. It is because the higher burning rate that makes the port area of last sub-grain expand faster and alle-



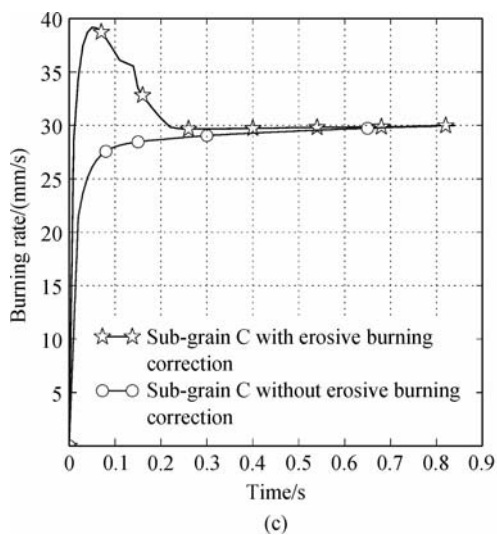


Fig. 9 Erosive and non-erosive burning rates profiles of sub-grains

viates the erosive burning effect.

As a comparison for analyzing the influence of internal ballistic time-dependent effect, an IBS of 0.001 s time step using quasi-steady model defined by Eq. (5) is conducted. As shown in Fig. 8(b), the trends of unsteady and quasi-steady model results are consistent, and the disagreements occur when pressure changes remarkably. During the initial pressure peak period, the time-dependent effect postpones the peak time and reduces the peak magnitude greatly. And it also make all transitional durations between the four distinguished pressure stages during the overall working period after initial peak. Therefore, the time-dependent effect on the internal ballistic performance of the tri-burning-rate SRM is significant and should not to be neglected in the simulation.

All the results above indicate that the erosive burning and time-dependent effect influence greatly the internal ballistic performance of multi-burning-rate SRM, and the simulation using unsteady internal ballistic model and applying erosive burning correction on the burning rates is necessary.

5 Conclusion

By analyzing the characteristics of multi-burning-rate SRM, a rapid simulation approach

coupled with 3-D multi-burning-rate grain regressing and time-dependent, lumped parameter internal ballistic calculation is realized. PFCADM method and reasonably simplified internal ballistic model have been proposed to raise the simulation efficiency.

Simulation using the method on a SRM of tri-burning-rate grain has been conducted. Quantitative agreement between experimental and simulated results has been obtained. Through the comparison of quasi-steady and unsteady models, with and without erosive burning correction of burning rates, it can be concluded that the erosive burning effect and the time-dependent factor have a significant impact on the internal ballistic performance of the multi-burning-rate SRM. It is also clear that the proposed internal ballistic numerical model can provide reasonable estimates of motor internal pressure and can predict the important initial pressure peak.

All the results indicate that proposed approach of multi-burning-rate SRM IBS based on PFCADM is capable of predicting the internal ballistic performance reasonably in acceptable computational efficiency, and it can also be applied in the preliminary designing of other SRMs with complex 3-D grain configurations.

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